CALCULUS.

220. Proposed by C. N. SCHMALL, College of the City of New York, New York City.

To determine the least polygon of n sides that can be described about a given circle.

Solution by the PROPOSER.

Let $\phi_1, \phi_2, \ldots, \phi_n$ be the successive angles contained between the lines $l_1, l_2, \ldots$, drawn from the center to the vertices of the polygon, and the radii (r) drawn to the points of contact of the sides. The area of the right triangle whose angle at the center is ϕ_1, will be

$$\triangle = \tfrac{1}{2} r l_1 \sin\phi_1 = \tfrac{1}{2} r . r \sec\phi_1 . \sin\phi_1 = \frac{r^2}{2}\tan\phi_1 .$$

Hence the entire area of the polygon is

$$\Sigma\triangle = \frac{r^2}{2}[\tan\phi_1 + \tan\phi_2 + \ldots + \tan\phi_n].$$

But $\tan\phi_n = \tan[2\pi - (\phi_1 + \phi_2 + \ldots \phi_{n-1})] = \tan[2\pi - \theta]$, where $\theta = \phi_1 + \phi_2 + \ldots + \phi_{n-1}$. Thus $u = \tan\phi_1 + \tan\phi_2 + \ldots + \tan(2\pi - \theta)$ is to be rendered a minimum. Differentiating partially with respect to ϕ_1, we obtain

$$\frac{\partial u}{\partial\phi_1} = \sec^2\phi_1 - \sec^2(2\pi - \theta) = 0.$$

Hence $\phi_1 = 2\pi - \theta = \phi_n$.

In like manner we may show that any angle equals the one preceding it. Hence the minimum polygon is regular.

Also solved by G. W. Greenwood, and J. Scheffer.

DIOPHANTINE ANALYSIS.

136. Proposed by A. H. HOLMES, Brunswick, Maine.

Given $7x^2 - 111 = y^2$. Required a value for y greater than unity which shall be a prime integer.

Solution by J. SCHEFFER, A. M., Hagerstown, Md.

Let $y=q$, $x=p$ be two values that satisfy the equation $y^2 - Nx^2 = -a$, and $y=n$, $x=m$ two values that satisfy the equation $y^2 - Nx^2 = 1$. Then we evidently have $y^2 - Nx^2 = (q^2 - Np^2)(n^2 - Nm^2) = n^2q^2 + N^2m^2p^2 - N(m^2q^2 + n^2p^2) = (nq \pm Npm)^2 - N(mq + np)^2$. Therefore, we can put $y = nq \pm Npm$, $x = mq \pm np$. Substituting numerical values we have, since $y=8$, $x=3$ satisfy the equation $y^2 - 7x = 1$,